

INVESTIGATIONS

- 7 Which of these numbers can be expressed either as a power of 3 or as a power of 4?

$$\frac{1}{9} \quad \frac{1}{24} \quad 64 \quad 12 \quad \frac{1}{16} \quad \frac{1}{27} \quad 24 \quad 81 \quad 36$$

- 8 Which of these expressions simplify to give the answer, a ?

$$\frac{5a^3}{a^2} \quad \frac{4a^3}{(2a)^2} \quad \frac{2a}{2a^2} \quad \frac{3a^{-1}}{3a^{-2}} \quad \frac{4a^3}{2a^2} \quad \frac{\left(\frac{a^{-1}}{25}\right)}{(5a)^{-2}}$$

- 9 A saver deposits £80 with a bank giving an interest rate of 5%. Use your knowledge of indices to express as $a \times b^n$ the value of the investment

a after five years

b after n years.

After how many years and months will the investment reach £100?

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- 6 A triangle has base length as given. The area is 6 m^2 .

Find the height giving your answer in the form $a + b\sqrt{5}$ where a and b are integers.



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- 6 Which four of these six lines would describe a square?

$$y = x + 1 \quad y = -x + 2 \quad y = x - 1$$

$$y = x - 2 \quad y = -x - 2 \quad y = -x + 4$$

Is it possible to find the area of the square?

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3 The equations of two straight lines are

$$ax + by + c = 0$$

and $dx + ey + f = 0$

- a Given that the lines do not intersect, write down an equation connecting a , b , d and e .
- b Given that the lines have the same y -intercept, write down another equation.
- c Given that the first line has identical x - and y -intercepts, write down a third equation.

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4 The two boxes below contain equations of straight lines. Each line in Box A is perpendicular to a line in Box B. Which line in Box B does not have a pair?

Box A

$$y = \frac{1}{3}x + 1$$

$$5y = -2x + 4$$

$$y = 3x + 2$$

Box B

$$y + 3x = 5$$

$$3y = -x - 1$$

$$2y - 5x - 2 = 0$$

$$x - 3y - 2 = 0$$

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8 Four equations and their solutions are given:

Equations:

1. $x^2 + 3x + 2 = 0$

2. $x^2 - 3x + 2 = 0$

3. $2x^2 - x - 3 = 0$

4. $x^2 - 4x + 4 = 0$

Solutions:

$x = -1$ $x = 3$

$x = 1$ $x = -3$

$x = -2$ $x = \frac{3}{2}$

$x = 2$ $x = -\frac{3}{2}$

Some solutions may be used more than once.

Match each equation with its solution(s).

9 The box contains some linear factors:

$(2x + 3)$ $(x - 2)$ $(2x + 1)$ $(3x + 1)$ $(x + 1)$ $(x + 4)$ $(x - 4)$ $(2x - 3)$ $(2x - 1)$

Use some of these factors to factorise the quadratics

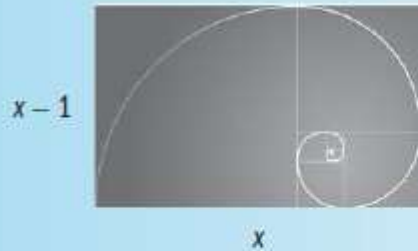
a $2x^2 - 5x - 12 = 0$

b $2x^2 - 3x - 2 = 0$

c $4x^2 + 4x - 3 = 0$

INVESTIGATIONS

- 10** The golden ratio is a mathematically interesting number that supposedly represents divine proportion. The golden spiral, occurring in nature, is formed from the golden ratio.

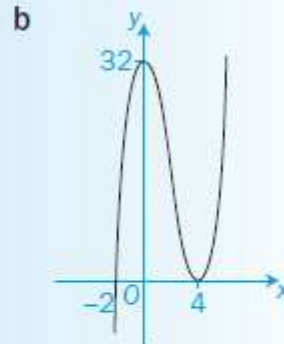
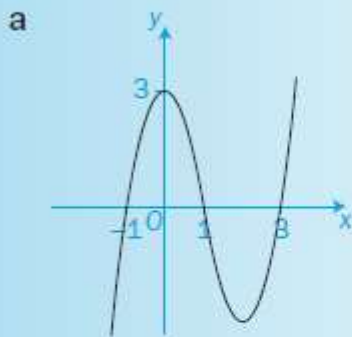


The rectangle framing the golden spiral has sides in the golden ratio if the area of the rectangle is equal to one.

- a Write an equation using this information.
- b Hence solve to find x , giving your answer in surd form.
- 11** Choose values of k to make each of these quadratic equations have just one real (repeated) root.
- a $x^2 + kx + 4 = 0$ b $x^2 + kx + 9 = 0$ c $x^2 + kx + 16 = 0$
- Can you generalise to a quadratic of the form $x^2 + kx + n^2 = 0$?
- 12** Which integer values for k result in the following equation having two real roots?
- $$x^2 + kx + 5 = 0$$

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- 9 Find the equation of each curve using the information shown in the diagram.



- 10 A quadratic graph can intersect with the line $y = 0$ once or twice.

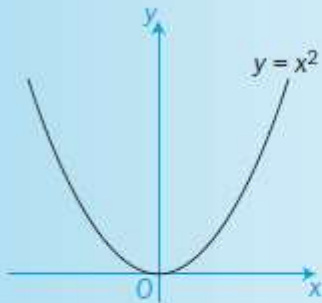
How many intersections are possible between

- a a linear and a quadratic graph
- b a linear and a cubic graph?

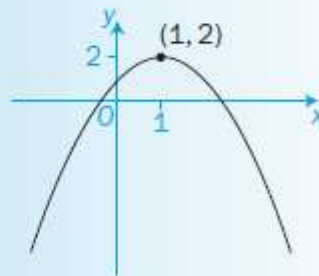
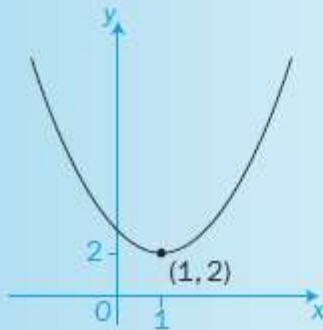
How could you prove your answers without drawing any graphs?

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11 The graph of $f(x) = x^2$ is shown:



Find the equations of the following graphs:



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12 Identify in which direction the following curves have been stretched.

- a $x^2 + 1$ \rightarrow $9x^2$
b $x^2 + 1$ \rightarrow $3x^2 + 3$
c x^3 \rightarrow $8x^3$

13 Given $f(x) = x^2 + 4$, match each description with its corresponding equation.

- a Stretch in the x -direction, scale factor = $\frac{1}{2}$ i $2x^2 + 8$
b Stretch in the y -direction, scale factor = 2 ii $4x^2 + 4$
c Stretch in the x -direction, scale factor = 2 iii $\frac{1}{2}x^2 + 2$
d Stretch in the y -direction, scale factor = $\frac{1}{2}$ iv $\frac{1}{4}x^2 + 4$

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6 Identify which of the following points

$(1, 4)$ $(1, 5)$ $(2, 0)$ $(2, 2)$

$(1, 6)$ $(1, 3)$ $(0, 3)$

would lie on the curve $f(x) = x^2 + 1$
after the following transformations.

a $f(x + 1)$

b $f(x) - 3$

c $3f(x)$

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10 A sequence u_n takes three consecutive integers, $n - 1$, n and $n + 1$, and multiplies them together.

Use logic to explain why the sequence gives terms which are always multiples of 3.

11 Look at the following sequence:

$\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$

a Find u_n .

b Now find u_n if the sequence has alternate positive and negative terms starting with $+\frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, \dots$

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- 9 a Which three consecutive integers add to give 300?
b Which four consecutive integers add to give 406?